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CRITICISM OF SPACETIME THEORY

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled CRITICISM OF SPACETIME THEORY submitted by RICHARD K. MABLY in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in THEORETICAL PHYSICS.

Abstract

The purpose is to examine spacetime theory in both classical and quantum contexts. The chronometric formulation of spacetime is rejected in favour of a constructive axiomatic approach. This begins with a discussion of which concepts should be used as primitive ones and whether locality is a desirable property of the axioms. The differentiable, conformal, and projective structures are constructed. These are then made into a Weyl geometry, in which geodesic clocks can be built. These in turn enable the space to be tested for metric structure; this is assumed to exist. Other aspects of axiomatics are mentioned. The theory is first criticized in a classical context; the weakness associated with the use of the real number continuum depends on an epistemological assumption. In the quantum context there is an immediate difficulty with the concept of particle. Some of the literature critical of spacetime theory is mentioned. In particular, there are limitations on the measurement of curvature. Properties of clocks are discussed. Some phenomenological considerations are presented and then it is suggested that clocks must be limited in accuracy by the Planck time. Implications of this limitation are discussed; the main one being that quantum theory lacks operational foundations. Experimental knowledge concerning a fundamental length is reviewed. Finally, brief comments are made about directions in which research is being pursued, namely on supergravity, superspace, and twistor theory.

Preface

The purpose of this essay is to present a critical analysis of the best available theory of space-time, which is GR. For a history of ideas about space-time, see Capek 76. The reader is assumed to know GR and quantum theory. GR is an abbreviation for general relativity; space-time is an abbreviation for space and time; spacetime is a four-dimensional differentiable manifold possessing appropriate structure. The format of references in the text is illustrated by this example: Ehlers PS 72 p.65 means page 65 of the article co-authored by Ehlers, Pirani, and Schild in 1972. The reading list contains uncited literature which is relevant to the subject of this essay.

I thank Martin Walker for encouragement.

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I. Construction of Spacetime Theory

.0 Introduction

We are going to discuss the foundations of physical theory. All physical theory is inferred from observations of nature. A theory is not fully appreciated unless these inferences are explicit. The axioms given here are the inferences made in the construction of GR.

.1 Chronometric approach

GR is a metric theory of space-time. Thus it would appear that we shall have to at some point discuss the measurement of distances and time intervals. Let us for a moment anticipate the question of how to perform such measurements; the reason for doing so is to dispense with the chronometric approach. The first method of space-time measurement that comes to mind is the everyday use of rulers and clocks. Can, in principle, all space and time intervals be measured using rulers and clocks? In the past the hypothesis had been accepted (Synge 60 p.107) that clocks can in principle measure any time interval. But the idea that rulers could measure all distances was rejected. The reason for the rejection of rulers is (in part) that to specify precisely what a valid ruler is involves some knowledge of the behaviour of matter (quantum theory, in fact), which in turn relies on the very theory of space-time to be constructed using rulers (Ehlers 73 p.34). This is a valid objection to their use. However it seems inconsistent to me that this same objection is not also applied to clocks.

After all, the clocks that their proponents had in mind also involved quantum mechanical behaviour of matter. Nevertheless it was postulated that in principle any time interval is measurable by some clock. It was then unnecessary to make an additional statement about the measurement of distances because this can always be done using clocks and light rays (see I.10). Clocks and rulers can be distinguished in this way: the separation of two events which lie in each other's light cone is directly measured by a clock; a ruler directly measures the separation of two events which lie outside of each other's light cone (Wigner 57 p.260).

Once the existence of clocks and particles has been accepted, in this approach the chronometric postulate is made (Synge 60 p.107). This states that whenever x and x+dx are "nearby" events experienced by a clock, their separation as measured by the clock is $|g_{ab}dx^a dx^b|^{1/2}$. It is assumed (a "zeroth postulate" is made) that space-time is describable by a four-dimensional differentiable manifold possessing a pseudo-Riemannian metric g of signature +---. To complete the foundations of GR in this approach the geodesic postulate can be made (ibid p.110). This is that the path of any freely falling non-zero rest mass particle in spacetime is that of a timelike geodesic of the symmetric affine connection compatible with the metric. And the path of a zero rest mass particle, in particular a photon, is a null geodesic.

.2 Rejection of chronometry

At this point, much of GR follows from the postulates. One can now ask, "how useful is this axiomatization?". (I make no distinction between axiom and postulate.) If one's aim is to find a set of axioms from which standard GR is easily deducible, then these axioms are of the right type (see Basri 66). But if one desires to begin from experience or as close thereto as possible, and via what are considered reasonable extrapolations construct a space-time theory, this system is terrible! It is this construction, rather than deduction, which I want to do, in order to see how and why GR is an inadequate description of nature.

Why is that axiom system so useless for our purposes? There are two reasons. First, the postulates simply "fall from heaven" (Ehlers PS 72 p.64). This is fine for a deductive axiomatics, but we wish to construct from experience as mush of the axioms as we can. With these two postulates there is little scope for understanding the reasons for their existence, or of how they might be modified. Second, by means of the geodesic postulate only, without the chronometric postulate, it is possible to construct clocks using particles and light rays (Ehlers 73 p.34). The chronometric postulate is redundant (or, with added interpretations, becomes more explicit than necessary). For these reasons we seek a better axiomatization of GR.

.3 Another approach

Is a better system known? Ehlers PS 72 and others have

given a useful characterization of GR by constructive axioms. (References to the literature are given by Ehlers 73 p.23.) I outline here the relevant part of the approach of Ehlers PS 72 and Ehlers 73 §2. Remember that our immediate goal is to construct axiomatically the theory of spacetime from concepts which are as primitive as possible. Space and time themselves might seem to be primitive concepts ideally suited to our task. But in fact they are not so. There are concepts closer to our immediate sensory perceptions than these, and from which these can be extracted, as shall be seen. Also, I think it would be difficult to give constructive axioms for space-time structure when space-time is accepted as an a priori concept.

From what else can we construct space-time theory? Clearly we will eventually have to include particles and light in this theory. They are basic elements of it and of our experience. Accept them as primitive concepts. A discussion of this acceptance appears in II. In what follows, quantum mechanics is completely ignored, except in I.11. From well motivated axioms about particles and light the whole theory of spacetime can be synthesized. Clocks can be made and even used to further investigate space-time structure (Ehlers PS 72 p.69, Castagnino 71).

.4 Primitive concepts

The primitive concepts we shall use are event, light ray, particle, and freely falling particle. The physical interpretation of our mathematical light ray is as a well-defined wave

packet such as a laser pulse or a gamma ray. By a particle we mean the history of a spherical non-rotating test particle such as a stone near the Earth's surface. The distinction between particle and freely falling particle is not necessary but is useful. It will be clarified as the theory is constructed. The important fact is that objects exist which satisfy the axioms of freely falling particles. The axioms which will be imposed using these concepts are to be interpreted as applying to regions of space-time which are empty except for the particles and light rays under discussion. We shall not consider regions occupied by other matter.

.5 Locality

Is it preferable that our axioms deal only with local properties of particles and light? (A local property is one which can be stated referring only to any given neighborhood of an event.) I think so, for the following reasons. Although the axioms are applied globally, local axioms are less unverifiable because most of our knowledge concerns our own neighborhood. Global axioms have a high degree of effectively intrinsic unverifiability. (See Ellis 75.) So with local axioms there is a better chance of arriving at a theory which is correct in our neighborhood, at least, regardless of whether this is sufficient to understand the whole universe. In any case, global axioms can always be added afterwards. (There are in addition philosophical aspects of this question.) The axioms I shall state, except axiom P, are local.

.6 Differentiable structure

M is defined as the set of all events. A particle is a subset of M. A light ray is a subset of M. $\mathcal{P}$ is the set of all particles. $\mathcal{L}$ is the set of all light rays. $\mathcal{P}_f$ is the set of all freely falling particles, and $\mathcal{P}_f \subset \mathcal{P}$.

Axiom P: Each particle has the structure of an oriented smooth manifold diffeomorphic to $\mathbb{R}$.

This is a large step towards a continuum theory. I shall refrain from specifying degrees of smoothness. The non-locality in this axiom is in the fact that it specifies diffeomorphism to $\mathbb{R}$. A local statement could only specify one-dimensionality. In the context of the full theory this axiom implies that there are no self-intersecting timelike curves.

Axiom L_1: For each event $a \in M$ there is a radar coordinate system $(x_a, U_a, P_a, P_a'; V_a)$ such that

(a) $a \in U_a$,

(b) $(x_b, U_b, P_b, P_b'; V_b)$ is a radar coordinate system and $U_a \cap U_b \neq \emptyset \Rightarrow x_a \circ x_b^{-1}$ is a smooth diffeomorphism.

This is equivalent to Ehlers' 73 p.24 formulation, which is as follows. There exists a family $F = \{(x_\alpha, U_\alpha, P_\alpha, P_\alpha'; V_\alpha)\}$ of quintuples $(x_\alpha, U_\alpha, P_\alpha, P_\alpha'; V_\alpha)$ with $U_\alpha \subset V_\alpha \subset M$; $P_\alpha, P_\alpha' \in \mathcal{P}$ such that

(a) x_α is a radar coordinate system with domain U_α based on P_α, P_α', relative to V_α,

(b) $M = \bigcup_\alpha U_\alpha$, i.e., $\{U_\alpha\}$ covers M,

(c) for each pair (α, β), x_α, x_β are smoothly related; and any radar coordinate system (x, U, P, P'; V) for M is smoothly

related to the x_αs.

A radar coordinate system (x, U, P, P'; V) is the following. For every event $e \in U \subset M$ there are precisely two light rays connecting it with each of the particles P, $P' \in \mathcal{P}$ (which are considered here as observers). As illustrated in figure 1, by such means four numbers can be assigned to each event e. The numbers are coordinates on the particles P, P'. The mapping $x: U \to W$ thus obtained is to be a bijection into the set $W \subset \mathbb{R}^4$, which is open in the usual topology. The choice of coordinates on P and P' is free; changes in coordinates on the particles induce, by axiom P, smooth radar coordinate transformations. Actually, we are not restricted to the use of radar coordinate systems only. Any smoothly related coordinates can be used. The set V is included in the definition to allow spacetimes possessing events which can be connected to particles by more than two light rays. (This can occur in a spacetime having compact space sections.) Thus a radar coordinate system relative to the subset V of M is one as described above for the sub-spacetime V with the restrictions of all particles and light rays to V.

Axiom L_1 gives M the structure of a four-dimensional differentiable manifold. The differentiable structure (largest set of smoothly related charts) containing the radar coordinate systems and the manifold topology which it induces, in which the sets U_a are open, will be used in the remainder of this construction. From this axiom we can view the four-dimensionality and differentiable structure of spacetime as a result of

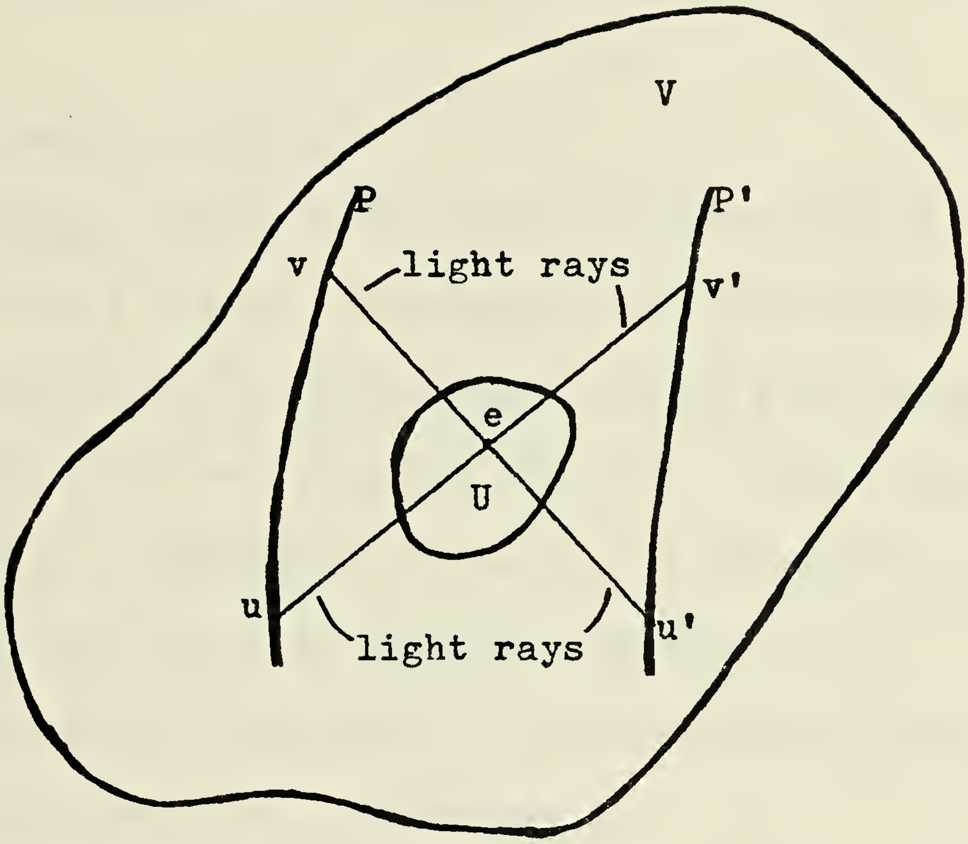


Figure 1. Radar coordinates (Ehlers 73 p.25)

the manner in which light rays and particles exist in space-time, particularly the fact that there exist particles which can "be seen by" and "see" events in a unique way. Note that no mensuration is involved here, only the assignment of quadruples of real numbers to events.

.7 Conformal structure

<u>Axiom L_2</u>: Every event e has a neighborhood V such that any event in V can be connected to any particle by at most two light rays within V. Furthermore, given a particle P containing e, there is a neighborhood $U \subset V$ of e such that any event $p \in U \setminus P$ can be connected within V to P by exactly two light rays, intersecting P in distinct events p_1, p_2. If t is a coordinate on $P \cap U$ and $t(e) = 0$ then $g(p) \equiv t(p_1)t(p_2)$ is a smooth function on U. See figure 2.

This axiom gives more detail about the relationship between particles and light.

<u>Axiom PL</u>: Particles and light rays are smooth paths (i. e., paths of smooth curves).

This axiom, with axiom L_1, relates the structure of each particle to that of others. The smoothness conditions in the two axioms seem to be necessary to the proofs that M has certain desirable properties. In particular, this axiom is needed to show that light rays are one-dimensional.

<u>Axiom L_3</u>: For every event e, $D_e \setminus L_e$ has two connected components, as does the set of vectors tangent to light rays.

The tangent space to M at e is denoted by M_e. A direction

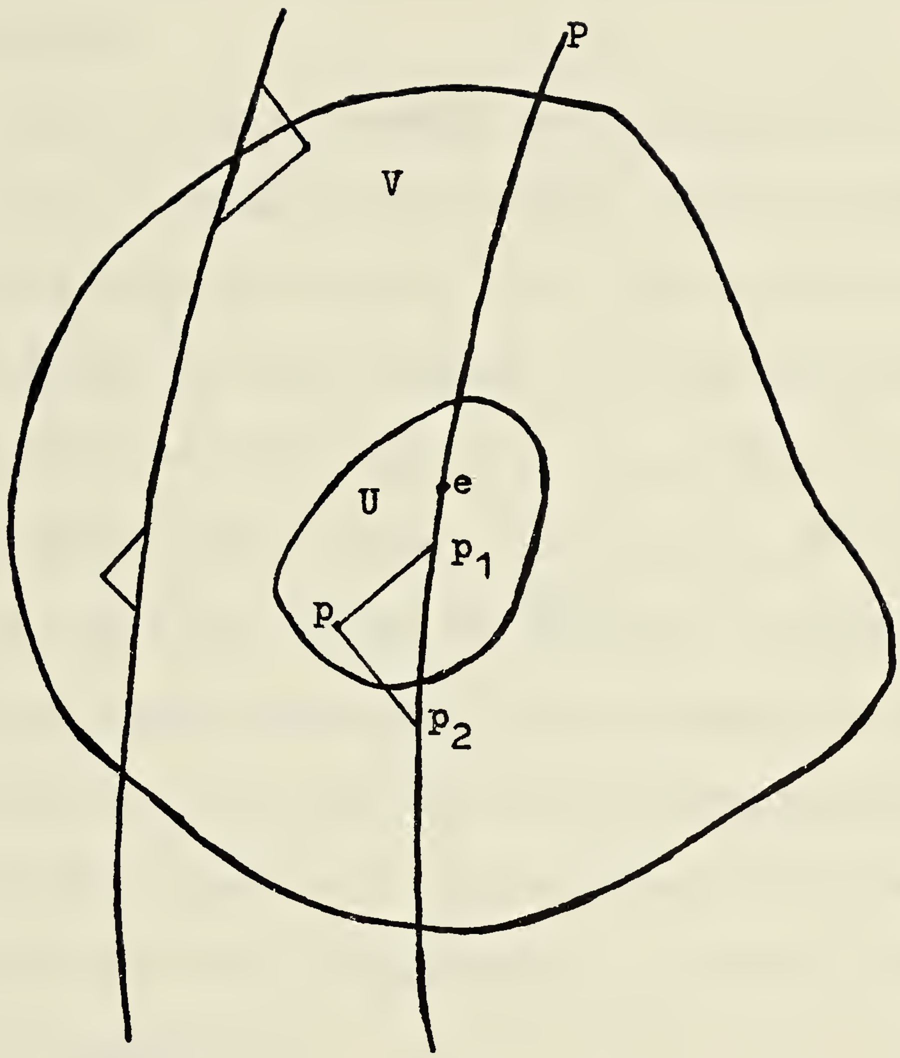


Figure 2. Axiom L_2 (Ehlers 73 p.26)

at e is a one-dimensional subspace of M_e. D_e is the set of directions at e and L_e is the set of light directions at e (generated by tangents to light rays). M_e has a natural topology which induces a quotient topology on D_e.

The individual concepts of space and time are now implicitly developed.

On M there exists a smooth pseudo-Riemannian metric g of signature +---, unique up to a smooth non-zero scalar factor, such that any tangent vector k to a light ray satisfies g(k,k) = 0. (It should be clear whether g in the following refers to the function or to the metric.) See Ehlers PS 72 p.72-74 for the proof of this theorem. As a result, M is endowed with a conformal structure, which is $\{\lambda g \mid \lambda(p) \neq 0 \ \forall p \in M\}$. There is thus also a causal structure on M. Special relativity is locally valid, in the sense that all statements referring to a single tangent space of M can be made as in special relativity. Examples are the orthogonality of vectors and the relative speed of two particles.

A hypersurface is called null iff its normal is null (in which case the normal is tangent to the hypersurface). A curve whose tangent is everywhere null and the path of which is contained in a null hypersurface is called a null geodesic. An easy lemma in conformal geometry is that given a null vector k at a point e, there exists a unique null goodesic through e having tangent k.

By the aforementioned theorem, a light ray is a null curve. I shall not show here that a light ray is a null geodesic.

However, the set $X_e \equiv \{p | g(p) = 0 \ \& \ p \in U\}$ (notation of axion L_2) of points near e connectible to e by a light ray "looks like a light cone in flat spacetime" in the sense that there are coordinates y^a in a neighborhood of e, smoothly related to radar coordinates, such that $g(p) = \eta_{ab} y^a(p) y^b(p)$, where η_{ab} is the Minkowski matrix. $X_e \setminus \{e\}$ is a smooth hypersurface. X_e is therefore termed the (local) light cone of e. V_e denotes the disconnected region defined by X_e (i.e., the "timelike interior" of X_e).

.8 Projective structure

Axiom F_1: Through a given event, in a given timelike direction there is a unique freely falling particle.

Axiom F_2: Each event e is contained in a permissible coordinate system y^a (said to be projective at e) such that every freely falling particle through e has a parametrization $y^a(u)$ satisfying $d^2y^a/du^2 \big|_e = 0$.

An equivalent statement of axiom F_2 which makes no reference to coordinates is desirable to make clearer the geometric content of that axiom. Such is not known to me.

These two axioms are an expression of the weak equivalence principle. They select freely falling particles for providing standard motions. Axiom F_2 implies that there is no invariantly definable gravitatiinal field strength at a point as there is in Newtonian theory; it also contains an infinitesimal version of the law of inertia.

ρ_f uniquely defines on M a projective structure such that

every freely falling particle is a geodesic. A projective structure is a set of locally defined symmetric affine connections the paths of the geodesics of which coincide. The statement is that these are paths of freely falling particles. As yet there is no preferred parameter for these particles.

.9 Weyl geometry

<u>Axiom C</u>: Each event e has a neighborhood U such that $\mathcal{V}_e = \bigcup\{P \mid e \in P \in \mathcal{P}_f\} \cap U \setminus \{e\}$.

Ehlers 73 p.31 states the condition as: an event $p \in U$, $p \neq e$, lies on a freely falling particle P through e if and only if p is contained in $\mathcal{V}_e$.

This axiom expresses a compatibility between the conformal and projective structures. $\mathcal{P}_f$ is characterized geometrically as the set of timelike projective geodesics.

It can now be shown (Ehlers PS 72 p.78-81) that light rays and freely falling particles define uniquely on spacetime a Weyl geometry, i.e., differential, conformal, and affine structures (a symmetric affine connection) such that light rays are conformal and affine null geodesics, and parallel transport preserves nullity (and hence orthogonality) of vectors. This fact justifies considering the sets $M, \mathcal{L}, \mathcal{P}_f$ as primitive elements of spacetime, in the context of the axioms we have imposed.

.10 Geodesic clocks

In a Weyl geometry what I shall call geodesic clocks can

be constructed from particles and light rays. A discussion of what constitutes a clock is given in II.9. To be fully rigorous the five-step construction which follows (taken from Ehlers 73 p.33-34) would have to use a limiting procedure, but I shall omit the mention of taking limits, for simplicity. The result of this geometrical construction is the affine parameter (defined only up to a linear transformation) along a freely falling particle P.

Figure 3 illustrates how a "line element" orthogonal to P is constructed. The diagonal line segments belong to null geodesics.

Figure 4 shows the construction of a "plane strip" containing P.

Figure 5, combining the two preceding ones, and omitting the auxiliary lines used in those, demonstrates a "comb". This is a set of orthogonal line segments contained in a plane strip with P.

A parallel to P is constructed as shown in figure 6. The particle Q will in general not follow a geodesic. Its existence is easily established.

Finally, by reflecting a light ray off P and Q a geodesic clock is obtained, as shown in the next figure. The light ray's intersections with P are equal parameter lengths apart.

In a spacetime region of negligible curvature, the measurement of the interval between two events p and q is performed as shown in figure 8. P is the world line of a clock. The separation is $|t_1 t_2|^{1/2}$. For more detail on such measurements

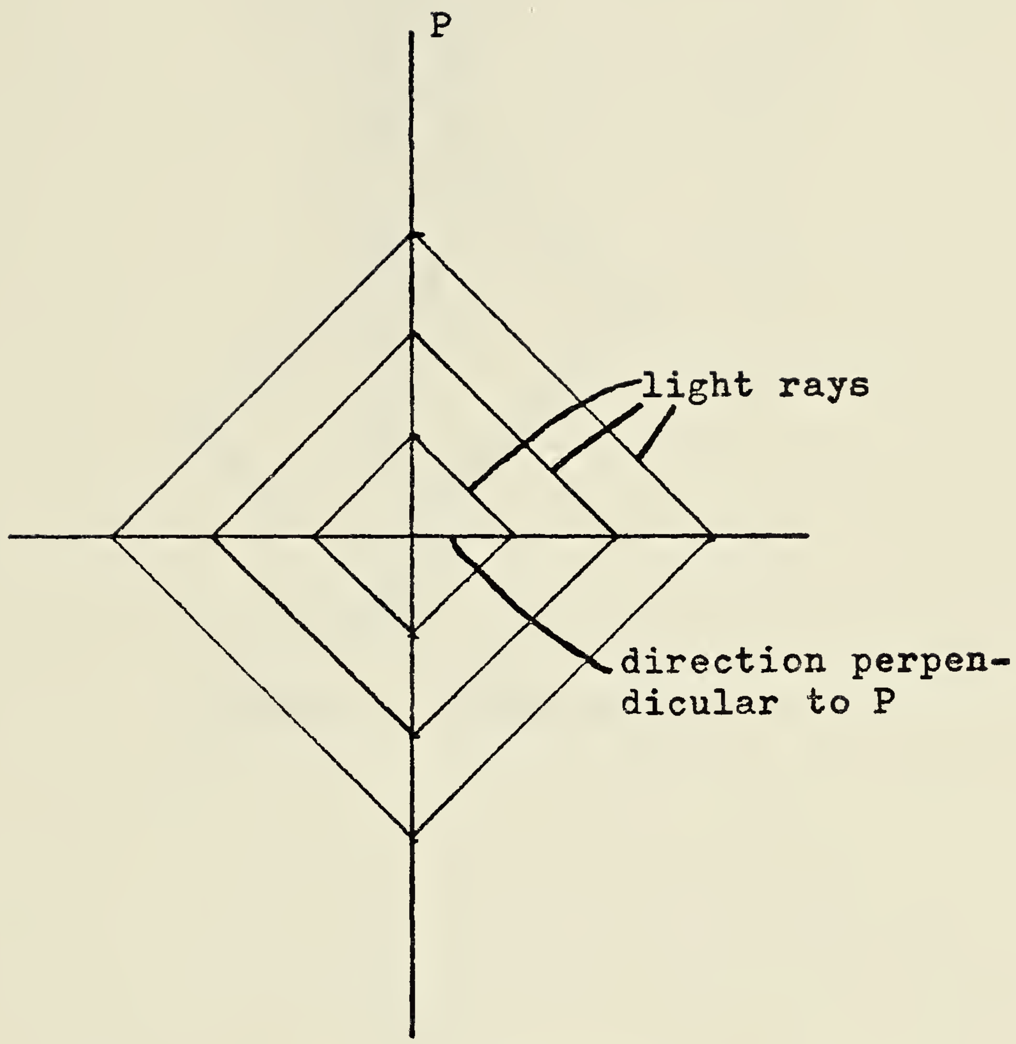


Figure 3. Orthogonality

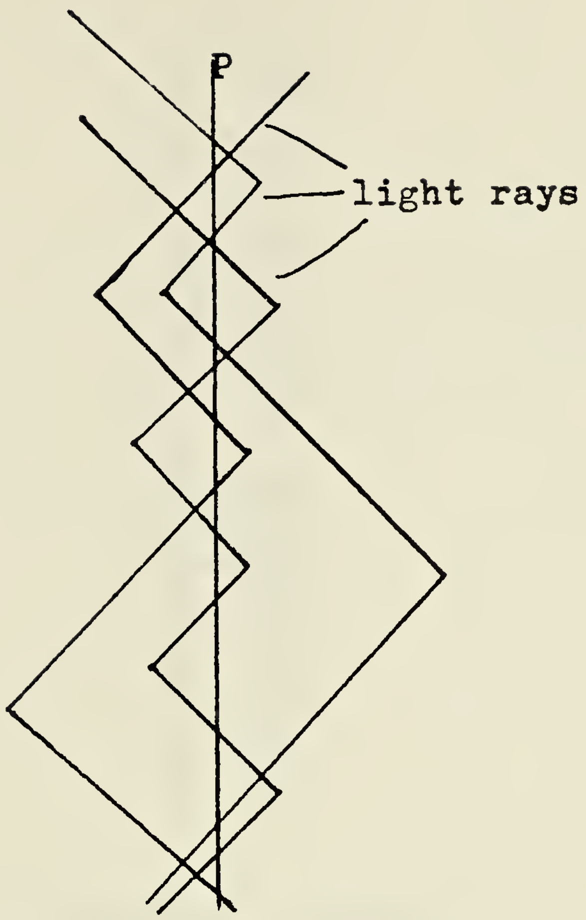


Figure 4. Plane strip

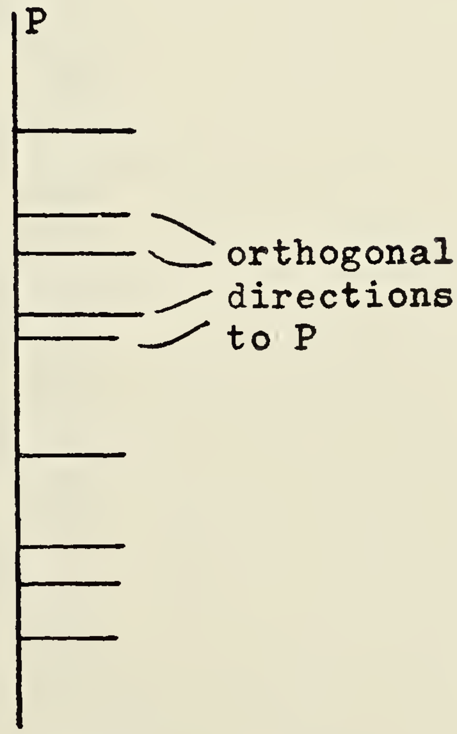


Figure 5. Comb

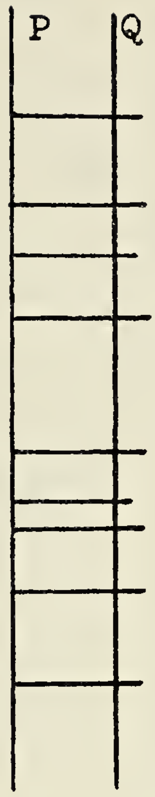


Figure 6. Parallel

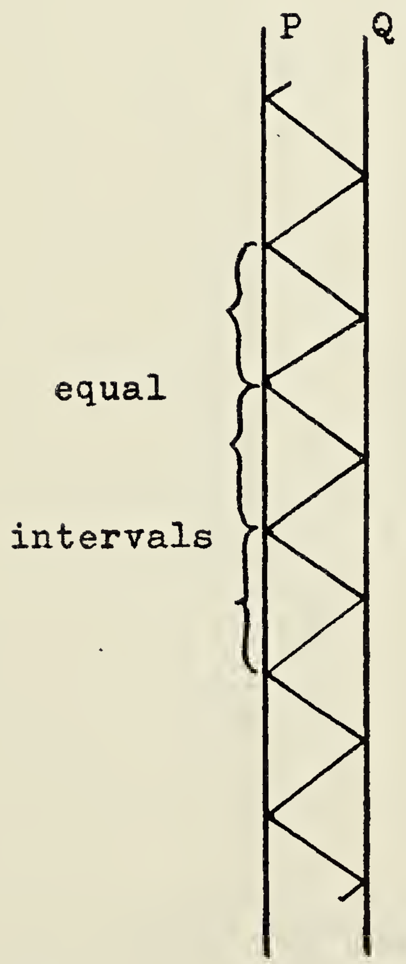
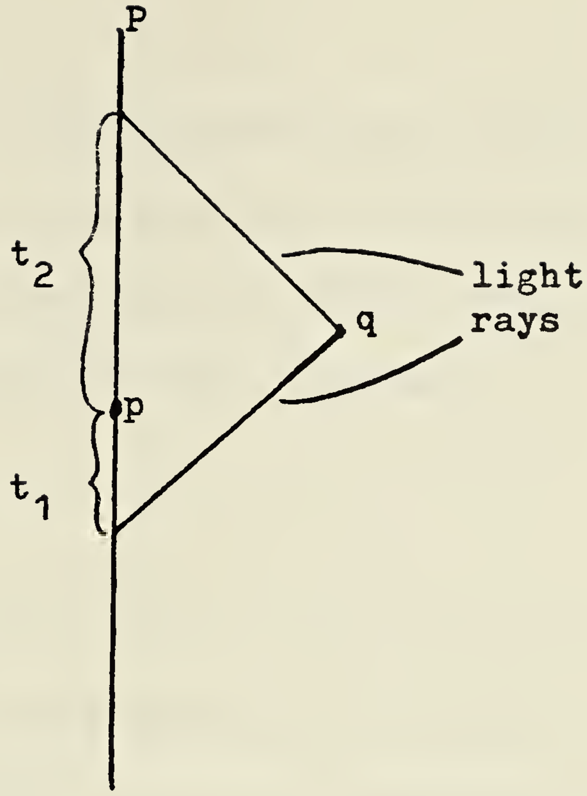
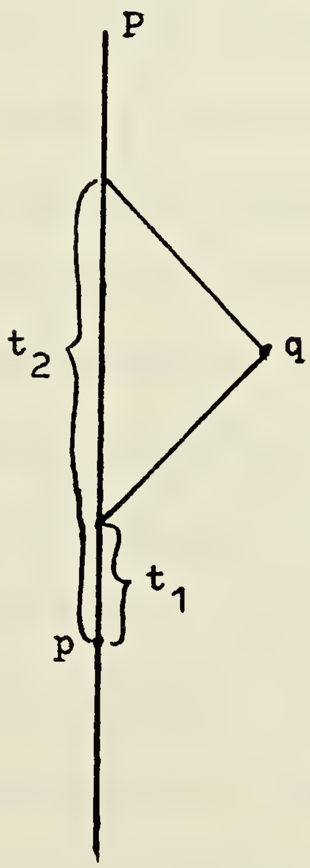


Figure 7. Geodesic clock



(spacelike separation)



(timelike separation)

Figure 8. Measurement of intervals

see Marzke W 64.

The geodesic clock brings us back very close to axiom P. The axioms we have stated allow us to geometrically characterize a dense subset of coordinates on P (as seen in II.1). This is of great importance in what follows.

.11 Metric structure

Geodesic clocks can be used in the next step in the axiomatization of GR, that of formulating the existence of a metric. The affine connection Γ^a_{bc} of the Weyl geometry naturally gives rise to a curvature tensor $R^a{}_{bcd} = 2(\Gamma^a_{b[d,c]} + \Gamma^a_{e[c}\Gamma^e_{d]b})$ which decomposes to $R^a{}_{bcd} = \hat{R}^a{}_{bcd} + \tfrac{1}{2}\delta^a_b F_{cd}$, where $\mathcal{g}_{e(a}R^e{}_{b)cd} = 0$, $F_{(ab)} = 0$, $\mathcal{g}_{ea}$ is a conformal metric (tensor density) and δ^a_b is the Kronecker delta. Consider parallel displacement around a loop at e, in the plane spanned by the vectors A^c and B^c. This induces a linear transformation in M_e, given by $C^a \rightarrow R^a{}_{bcd}A^cB^dC^b$. Since parallel displacement in a Weyl space preserves nullity of vectors, the transformation must be the product of a Lorentz transformation, generated by $\hat{R}^a{}_{bcd}A^cB^d$, and a dilatation, generated by $\tfrac{1}{2}\delta^a_b F_{cd}A^cB^d$.

When does a pseudo-Riemannian metric g_{ab} exist, which is compatible with the conformal structure and whose Levi-Civita connection is the Weyl connection? (It is clear that these two conditions should initially be required of a metric.) The equations of the preceding paragraph are still true if $R^a{}_{bcd}$ is the curvature tensor of the Levi-Civita connection Γ^a_{bc} of such a g_{ab}, because of the first condition on g_{ab}.

The second condition implies that no dilatation of a vector can occur in parallel transport around a loop using the new connection Γ^a_{bc}. Otherwise the scalar product of a non-null vector with itself would change on transport around the loop, which is impossible. Thus a metric satisfying the stated conditions exists iff $F_{ab} = 0$. Using a geometrical definition of congruence of vectors and parallel transport (given by ibid p.34), one can ask whether a time unit when parallel transported from one event to another along different paths gives differing lengths of time. Our experience with the clocks defined by atoms and smaller objects, e.g., via their Compton periods h/mc^2, indicates a negative answer. Such objects appear to have constant ratios of masses, lifetimes, and transition frequencies. We have little experience with geodesic clocks. If it is assumed that "atomic time" (as given by the aforementioned systems) is identical to geodesic clock time then we must put $F_{ab} = 0$. The resulting metric is unique up to a constant positive factor.

<u>Axiom R</u>: $F_{ab} = 0$. Equivalently, there is a pseudo-Riemannian metric which is compatible with the conformal structure and whose Levi-Civita connection is that of the Weyl geometry.

.12 Remarks about axiomatics

The axiomatization continues. We shall not follow it farther because for the purposes of this essay we are in possession of as much of it as needed. Let me remark, however, that although I think the Ehlers PS 72 system is the best available,

others have been attempted. As mentioned earlier, Ehlers 73 gives references to some of the literature. So does Reichenbach 69 p.x. In particular if the work of Woodhouse 73 were to be extended, it could be quite useful. His approach gives a more fundamental role to causal relationships in space-time than to topological structure. (The study of Zeeman topologies is relevant in this regard; see Gobel 76.) Nevertheless, the nature of the considerations presented here is such that the conclusions will, I think, be durable.

II. Criticism of the Theory

.0 Contexts

I now undertake to criticize the foundations of space--time theory as represented by the preceding axioms. There are several contexts in which they can be analyzed, including classical, quantum, and philosophical ones. Figure 9 schematically depicts logical and conceptual relationships among fundamental physical theories. The conceptual development of (relativistic) quantum theory from (special) relativistic mechanics seems to me to be totally unrelated to the development of GR from Newtonian theory, and I have indicated this by drawing their directions of development orthogonally. There is no question that special relativity logically precedes relativistic quantum theory. One could perhaps question the indicated relationship between special and general relativity, though. Anyway, we shall first briefly examine the axioms in a classical (i.e., their own) context. Then, as indicated by the figure, they must be analyzed in the context of the larger theory built on them, namely, quantum theory.

I shall assume that the axioms are self-consistent.

.1 Classical context: first axiom

Are the primitive concepts understandable? On the macroscopic level there is no problem. There is nothing to indicate at this stage that they are inextendible to a microscopic level, and thus they are accepted. (There is no natural length or time scale in the theory which would hint at inextendibility.

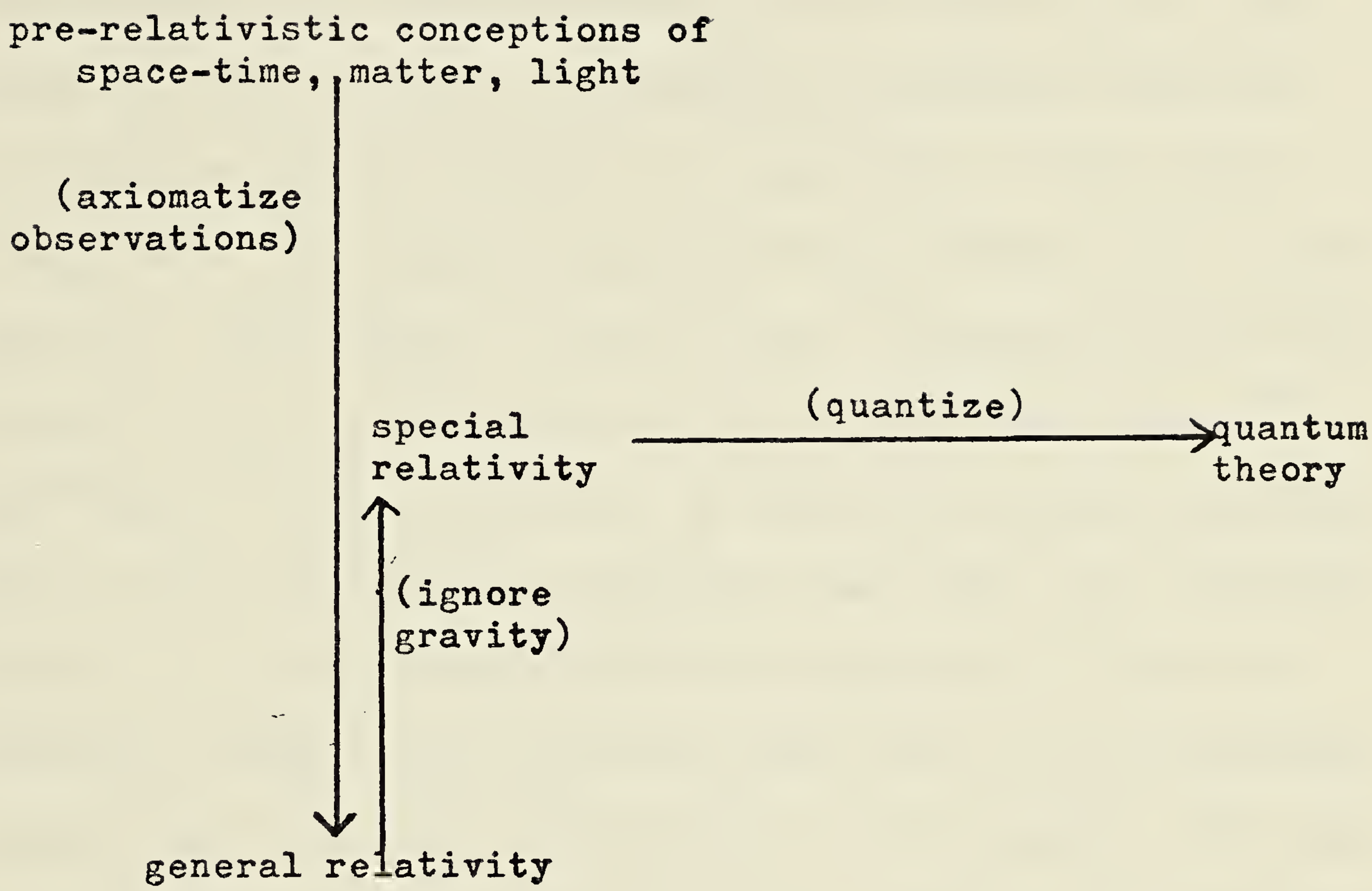


Figure 9. Logical relationships

For the same reason, there can be no absolute measurement limitation.)

How verifiable is axiom P? Not compl̶etely, even excepting its global statement (homeomorphism with $\mathbb{R}$). I don't think the requirement of orientation has any physical content. The existence of local coordinates can be tested (i.e., disproved but not proved) by attempting to construct sequences of clocks as shown in figure 10. Begin with a clock (call it Q_o) which uses a given freely falling particle P and a particle Q_o. This defines a unit of time. Construct successive clocks Q_1, Q_2, ..., the interval of the nth clock being 2^{-n}. $\{m2^{-n} \mid m \in \mathbb{Z},\ n \in \mathbb{N}_o\}$ is dense in $\mathbb{R}$ (with the standard topology). Thus if this construction failed for some $n \in \mathbb{N}_o$, the continuum nature of a particle would have to be rejected. It must be admitted, though, that a particle could fail to be a continuum without that fact being demonstrable by this method. The jump to a continuum theory is made for its mathematical convenience. We see that the first axiom is weak, even classically!

.2 Epistemological assumption

I must mention that I think that this defect results from an assumttion which is implicit in our reasoning and which axiom P almost directly contradicts. A somewhat vague statement of this assumption is that human beings can count, but can't "continue". That is, countability is an undeniable part of our experience, but the continuum is farther away or even unnatural. (Einstein claimed "that measurements can ultimately

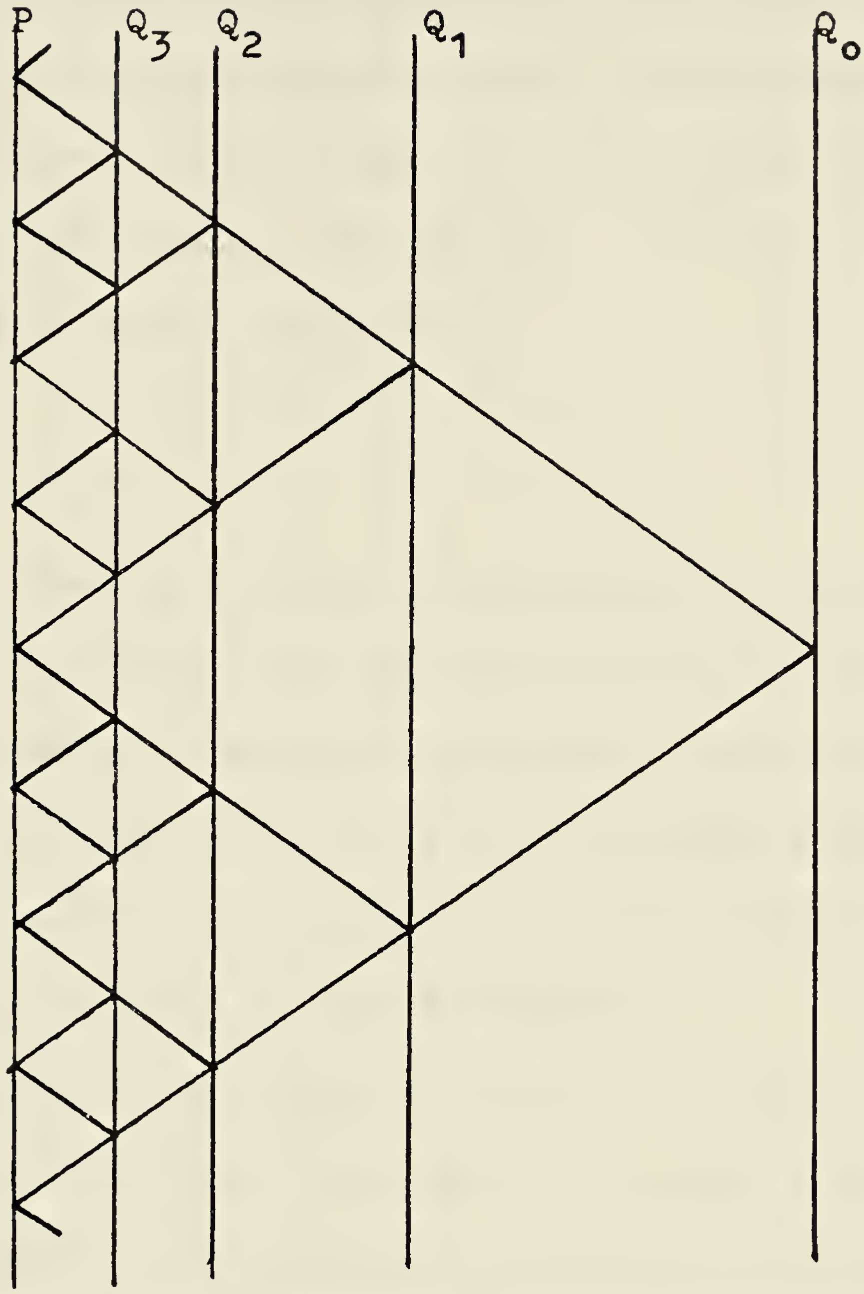


Figure 10. Affine parameter

be reduced to the observation of coincidences and counting"; Ehlers 73 p.54.) Therefore any physical theory which uses the continuum must be defective (although not necessarily incorrect). At present it is necessary to allow such a weakness for the sake of mathematical convenience. (For more on this see Penrose 66 p.8-9, 72 p.333-334.)

.3 Remaining axioms

I shall not examine all the remaining axioms individually. But it is easy to see that the weakness of axiom P reverberates throughout the system, each axiom reflecting the weakness of the first one. However, aside from the continuum weakness, I think that this system is strong. Ehlers 73 p.28,30 mentions how axioms L_1, L_2, L_3, F_1, F_2 can be tested.

The Riemannian axiom (axiom R) can be justified only on quantum mechanical grounds at present. "It seems that the existence of accurate clocks is deeply related to the quantum nature of matter": Penrose 68 p.129. In principle, though, testing of axiom R can be achieved through the use only of geodesic clocks, so its weakness reduces simply to the fact that this has not been done. See also II.5.

.4 Quantum context: particles

It is on the quantum context that I shall concentrate, for this has bearings on the direction of future theory. What do I mean by the quantum context? The answer is apparent in what follows. I evade this question because the quantum con-

text is not especially well defined. Relativistic quantum theory is built on special, not general, relativity. There is no quantum theory of or in general (meaning curved) spacetimes. This fact will not deter me from applying "quantum ideas" to the examination.

Begin with the same two questions. Are the primitive concepts understandable? Are the axioms self-consistent? (This was left implicit earlier.) Widening the context of considerations cannot alter the self-consistency of a given set of axioms. But the second question is included here to emphasize the difficulty which is created by the first question anyway. If one includes in "the axioms" not only those of spacetime theory but also those which build quantum theory on top of it, then the answer to both questions is no! On the one hand, a classical idea of particle is embodied in axiom P in order to construct spacetime, while on the other hand, a central fact of quantum theory is that the classical idea of a particle is wrong, except in certain approximations. There is a stark contradiction here. But this is actually the position of present theory! Particle is an acceptable primitive concept in some circumstances, but not in all. (Although this situation has existed for close to fifty years, only recently have more than a few physicists become seriously concerned with it.)

(The view that quantum theory and GR can exist harmoniously, each true in their separate domains, with no need to unify them in a theory in which quantum and gravitational effects

are simultaneously important, is untenable from a physical as well as a logical standpoint; for a discussion of the latter, see Wigner 57 p.260-263.)

.5 What to do about particles

How can this obstacle be removed? The only way I can think of in which matter is to retain its quantum behaviour is by starting afresh the construction of space-time theory, in a way that will be consistent with quantum properties of matter. This supports Penrose's 68 p.132 view that it may be mistaken to (attempt to) make one of space-time theory and quantum theory logically antecedent to the other. It will become clear (if it isn't already) that GR cannot result from such a program. In fact I think it is unlikely that any continuum theory could ultimately survive.

To make progress now, I shall ignore the problem, staying with GR and adding a little bit of quantum theory. I retain GR because only it comes close to an operational definition of event, which is the other primitive concept that I will attempt to show is unsatisfactory. The difficulties in the use of event as a basic concept are not as obvious nor as widely appreciated as those for particle. Light rays as primitive concepts come through this examination almost unscathed.

.6 History

First let's dispense with history. In the first paper on quantum theory, Planck 1899 noted that using the three con-

stants G, c, h, (gravitational constant, speed of light, Planck's constant) a quantity with the dimension of length could be formed. (Hereafter I denote this Planck length by $L = (hG/c^3)^{1/2} \cong 4.0 \times 10^{-33}$ cm. The word Planck followed by a dimension will denote the quantity of that dimension formed using G, c, h.) It appears that there was very little thought about the existence of a fundamental length in nature until Heisenberg 38, 43 published his ideas. The divergences in quantum field theory and the fact that a length might be able to remove them stimulated thought on the subject. In 47 Snyder published the beginning of a theory of quantized space-time. His method was recently still in use (Tamm 66, Kirzhnits C 68). Another attack on the continuum was made by Schild 49. Since that time, and especially in the last fifteen years, there have numerous philosophical and semi-philosophical attacks on the continuum, often by eminent physicists. One of the claims I make in this essay was made but not substantiated by van Dantzig 56 p.52, when he said that "physics does not provide us with any means of defining empirically the elements of space-time, ... i.e., possible events with coordinates to be defined with infinite accuracy". However about fifteen years ago an important change occurred. Formerly since the fundamental length was to remove divergences in quantum field theory, it was believed that its order of magnitude might be 10^{-14} cm. With the increase in interest in GR, the Planck length was resurrected, and is nowadays seriously thought to be a fundamental length (Misner TW 73 p.1193,1215). The re-

appearance of the Planck length is also due to questioning of the operational foundations of GR in a quantum context. The earliest paper on this of which I am aware is by Osborne 49, in which the Planck length and mass appear. This paper, though, seems to have attracted little attention. Wigner 57 and Salecker W 58 stimulated thought by investigating properties of clocks. "A Possible Connection Between Gravitation and Fundamental Length" was claimed by Mead 64. Nowadays many physicists think that the differentiable manifold is a "highly fictitious concept" (Ehlers 73 p.37) in physics, or as Penrose 72 p.334 puts it, that "'points' have actually very little to do with physical reality".

.7 Limited measurability of curvature

In applying quantum ideas to the concept of event one quickly thinks of the uncertainty principle. The papers I have seen on this all apply the uncertainty principle; an argument which does not use it will be given later. Osborne's 49 reasoning is as follows. The aim here is not to test the concept of event, but rather to find out under what conditions spacetime curvature is operationally defined. Let us try to measure the curvature (necessarily averaged over some region) of the Schwarzschild field. If $\{e_1, e_2, e_3, e_4\}$ is an orthonormal tetrad then $M_{\alpha\beta} \equiv R_{ijkl} e_\alpha{}^i e_\beta{}^j e_\alpha{}^k e_\beta{}^l$ is called the sectional curvature of the space spanned by $\{e_\alpha, e_\beta\}$ or geometrically, the Gaussian curvature of the surface formed (locally) by geodesics having tangents in the span of $\{e_\alpha, e_\beta\}$. R_{ijkl}

is the Riemann tensor. In the Schwarzschild solution with the usual coordinates, the leading terms in $M_{\alpha\beta}$ are of the order of $(GM/c^2r)r^{-2}$ and $(GM/c^2r)^2r^{-2}$ where M is the mass of the source. The error in the curvature must be much less than each of these two quantities. The curvature of a surface is given geometrically by $K = (\varphi_1 + \varphi_2 + \varphi_3 - \pi)/S$ where φ_a are the angles and S is the area of a geodesic triangle, in the limit as the triangle shrinks to a point. Consider the case in which $GM/c^2r < 1$ and the two-surface specified by e_α and e_β is spacelike. (Other cases lead to no stronger restrictions.) Since the Schwarzschild solution is static, particles moving along appropriate curves can be used to find angles and area. The uncertainty in the curvature measurement is $\delta K \sim \delta q/q^3$ where q is the linear dimension of the triangle and δq is its uncertainty (uncertainty in the position of a particle used to measure q). The uncertainty principle and other measurement limitations combine to give $(c^2r/GM)(ch/GM^2) \ll 1$ as the condition that the curvature is measurable. Combining this with the initial inequality, $GM/c^2r < 1$, implies the following: $M \gg M_* \equiv (ch/G)^{1/2} \cong 5.5 \times 10^{-5}$ gm. $r \gg L$, which is much graater than the Compton wavelength of M (a reasonable restriction), by the first condition. And $r/L \ll (M/M_*)^3$. These results imply, in particular, that the gravitational field of an isolated spherical body of mass less than M_* which has no angular momentum (a spinless elementary particle, for example) is operationally undefined. In terms of a continuum theory of quantum gravitation, people say that these limitations arise

due to quantum fluctuations of the metric on the Planck scale of dimensions. So while Osborne has exposed no flaw in the concept of event, he does suggest that "the curvature of space [time] should arise as a statistical concept valid only for very large numbers of particles", and concomitantly, that spacetime cannot be considered as curved at each event, but only over regions of extent at least L. L then is fundumental, but so far only to gravitation.

.8 Mead's work

The work of Mead 64 is in two independent parts. In the first he "deals with the question of whether present physical ideas about gravitation, together with the uncertainty principle, are sufficient to lead to a fundamental length", where such a length is one specifying "a limitation on the possibility of measurement". The second part intends to show an equivalence between the existence of a fundamental length and quantum fluctuations of the metric.

The whole first section is based on the fallacy that "the physical interpretation of the general theory of relativity requires that $g_{oo} = 1 + 2\phi > 0$" (ibid p.B851). Here g_{oo} is the tt component of the Schwarzschild metric in usual coordinates and $\phi \equiv -GM/r$ (with $c = 1$). His claim is thus that the $r < 2GM$ region of Schwarzschild spacetime is unphysical. To be sure, there may be grains of truth in some of what he claims; but they can only be reached via other reasoning. In an appendix Mead purports to show using completely general facts from

quantum mechanics that two clocks cannot remain "synchronized with the 'world time'" (ibid p.B861) with greater accuracy than L/c. However, I cannot understand the appendix. Nevertheless, I believe that he did not intend to make a strong attack on the concept of event because he says that a single measurement can be made with arbitrary accuracy.

The second part of the paper gives a heuristic argument for the previously stated equivalence, but the physical interpretation implied is somewhat obscure, partially due to the use of the word "fluctuations". In any case, I think it is obvious that an absolute space-time measurement limitation implies limited measurability of the metric.

Murphy's 74 argument for an absolute space-time measurement limitation is also flawed.

.9 Properties of clocks

What are the minimum mass and minimum mass uncertainty allowed by general quantum mechanical principles for a clock able to measure time intervals of length T with an accuracy τ? This is the question posed by Salecker W 58. First of all, they recall that von Neumann pointed out that a measurement (useful to humans) is not completed until its result is observed by some macroscopic object. They therefore break a clock into two parts: a discriminator and a signal. The former can be a microscopic device capable of discriminating between events having time separation greater than τ, and determining the separation up to T. The signal is a few quanta

sent from the discriminator to a macroscopic observer. It is the signalling which imposes limitations on the clock beyond those of the uncertainty principle. Under an idealization to one spatial (and one temporal) dimension they derive two possible restrictions on the mass M of the clock. If the position of the clock is allowed "to introduce a statistical element into the measurement of time" (ibid p.573) then $M > \hbar/(c^2\tau)(T/\tau)^{1/2}$. If not, $M > \hbar/c^2\tau(T/\tau)$. They believe but were not able to show generally that requiring the clock to have a small size imposes further restrictions on its properties. In the second half of their paper, examples of microscopic clocks are given.

The work of Salecker and Wigner does not call into question the concept of event, for two reasons. First, the mass restriction only says that to measure a given time interval a certain amount of mass is required. (Just for fun, a "mass of the universe" equal to 10^{80} proton masses gives a minimum τ less than 10^{-100} sec.) Second, the introduction of macroscopic considerations is foreign to the issue. We should not and shall not be concerned with signalling mechanisms; the geodesic clock has none.

We have now essentially exhausted the literature of thoughts on the measurability of events in a quantum context. However I will present a few more ideas for consideration.

.10 Phenomenologically based considerations

Any measurement of time must involve at least one of the four known interactions (strong, electromagnetic, weak, gravi-

tational), if there are no others. The first three have "characteristic times", which are respectively (Perkins 72 p.21) 10^{-23} sec., 10^{-16} sec., 10^{-8} sec. The strong interaction characteristic time is the order of magnitude of the pion Compton time. I do not know, though, whether these times are "really explained" by theory. Regardless of that, if one accepts them as part of quantum theory and they provide an approximate lower limit to the time required in their respective interactions, then one can infer that they limit time measurability in principle. For example, the geodesic clock uses photon reflection. This reflection requires at least about 10^{-16} sec., and hence the accuracy of the clock is limited. I did not mention a characteristic time associated with gravitation. The three characteristic times given above are related inversely to the strength of the interaction, so it seems that the gravitational time should be very long. But the only known time intrinsic to (quantum) gravitation is the Planck time which is approximately 1×10^{-43} sec. Anyway, we are really interested only in the existence of such a time. Summarizing the above argument, we can say that if quantum theory includes only the four known fundamental interactions and there is a lower bound on the time required for each, then in principle, events are unobservable. The argument is weak because of its heavy dependence on phenomenology; but it does tell us something.

Another phenomenologically based argument against events is given by Blokhintsev 73, where the "quantum principle"

which is added to GR is that there is a finite upper bound to the rest masses of particles.

.11 Reexamination of geodesic clocks

Let us examine the geodesic clock again and apply the first known quantum principle to it in order to derive an absolute measurement limitation. Suppose that we require a clock which can measure times as small as τ along a freely falling particle P. The clock consists of a zero rest mass particle Z bouncing back and forth between P and a particle Q a fixed distance away (i.e., following a parallel world line). What is a "zero rest mass particle" in GR? It is a localized wave packet of a field on spacetime. The points of reflection of Z on P and Q must be well defined for the required accuracy to be attained. The facts that the reflection points must be no more than τ apart and that Z is a wave packet being reflected by a particle imply that the (central) frequency of the wave must be at least $1/\tau$. This much is classical.

The quantum principle $E = h\nu$ tells us that an energy at least h/τ is contained between P and Q. The application of the quantum principle to an electromagnetic wave packet in flat spacetime can be interpreted as follows. Specify a wave packet to which a representative frequency ν can be assigned. (Just how the latter is done I am not sure.) Find its energy E_o according to the classical expression (integral of sum of squares of fields over all space). The physically allowed wave packet of the given shape has amplitude $(h\nu/E_o)^{1/2}$ times

that of the original.

The spatial separation of P and Q is no more than $c\tau/2$ (as measured by P). In three dimensions, we can put a sphere of radius $c\tau$ around the clock. More precisely, this sphere could be the exponential of the set of tangent vectors orthogonal to P and having length $c\tau$. Thus there is an energy h/τ contained in s sphere of radius $c\tau$, provided that there are no negative energy sources. Use Thorne's 72 p.237 "eminently reasonable conjecture: Horizons form when and only when a mass M gets compacted into a region whose circumference in EVERY direction is $\mathcal{C} \lesssim 4\pi GM/c^2$". Thus a horizon forms around a geodesic clock when $c\tau \lesssim 2Gh/(\tau c^4) \Leftrightarrow \tau \lesssim L/c$. Accepting Thorne's conjecture, I think that it is even more "eminently reasonable" to believe that the geodesic clock ceases functioning within the time τ, i.e., before one unit of time has been measured.

What have we done? We have shown that geodesic clocks are unable to measure intervals with accuracy better than L/c. There are weak points in the argument, though. It contains imprecise statements: references to energy and mass. It relies on conjecture, the proof of which may require further axioms embodying higher structure. We want to avoid requiring much structure because the breakdown of the theory can then be attributed solely to the higher structure, such as the field equations. (The metric structure plays an essential role in defining distances. Without it, the theory possesses distance information only topologically, which can never lead to an absolute measurement limitation.) In spite of its weaknesses,

I think that the argument is physically as plausible as can be.

.12 Generalization of argument

If the argument above is accepted, is the concept of event operationally defined? The answer to this question depends on how intrinsic (or canonical) the geodesic clock is perceived to be to the theory. However we need not debate this question, for the argument generalizes to any device built from the concepts accepted in our axiomatization which could be called a clock. Such a device is an approxiaation to a geometrical realization of axiom P. Thus clocks (in the limit mentioned in II.1) embody all operationally defined properties of particles. This is why our arguments using clocks can be applied directly to the axioms.

Clearly the "ticking" of the clock must be given by intersections of particles and light rays (Wigner 57 p.260); it being necessary to use particles, as recalled by Marzke W 64 p.48. For the clock to have an accuracy τ, these intersections must be well defined on such a time scale. This implies that the Compton period $h/(mc^2)$ of a particle of rest mass m must be less than (or of the order of) τ and that the period of a massless particle must be less than τ, as stated before. The energy of such objects is at least h/τ. During a time τ the particle or massless particle is contained within a sphere of radius $c\tau$. As reasoned before, the clock ceases to function within one time unit of its construction, before it has

measured anything, when $\tau \lesssim L/c$. Of course, this argument has the same weaknesses as the original one. We have also invoked another quantum principle: that a massive particle of mass m behaves like a wave of frequency at least mc^2/h.

.13 Ways around limitations

Could not a consistent and complete (in some sense) application of quantum theory to the problem of measurement extricate physical theory from the dilemma? For example, Hawking 71 p.76 says that "Since gravitational collapse is essentially a classical process, it is probable that black holes could not form with radii less than the Planck length ... the length at which quantum fluctuations of the metric are expected to be of order unity". However the whole problem is, in one aspect, just the fact that no consistent and complete quantum theory in curved spacetime is known. That is also why it is difficult to make precise my argument for unmeasurability. Another way of stating the problem is to ask for a consistent constructive axiomatization of quantum theory, including the space-time theory on which it rests. This has not been done even with the neglect of gravity.

Can the use of gravitational waves not obeying $E = h\nu$ circumvent the breakdown of geodesic clocks? No. The generalized argument still applies, and the particles which must be used in the clock will cause it to break down.

Harvey 76 mentions difficulties in the practical realization and use of geodesic clocks. I am concerned only with

their realizability in principle.

.14 Implication for axiom L_1

I remarked that space-time acquires a differentiable structure through axiom L_1 postulating a particular relationship among events and particles. Part of the relationship is what Ehlers 73 p.24 calls the "universality of light propagation": this being the independence of light propagation of its frequency, polarization, and intensity, and of the motion of the source. However, what we have discussed makes the validity of this idea at least questionable. It seems that photons of energy greater than the Planck energy might not exist. In axiom L_1 there was also the fact that particles exist which can "be seen by" and "see" events in a unique way. But apparently events which are very close to a particle cannot "be seen". Axiom L_1 therefore, while being local, must not be "too local". Really, it is false in the context of quantum theory. There is a hint here of a connection between fundamental (i.e., "small") particles and the Planck length.

.15 Quantization of the gravitational field

A question I posed when I started this investigation is "Are there incompatibilities or contradictions between GR and quantum theory?". There certainly are, as we have been seeing; the incompatible concepts of particle is a glaring example. Another is the dilemma mentioned in the previous paragraph: axiom L_1 is contradicted by quantum theory. I asked that

question as a precursor to "Must the gravitational field be quantized?". DeWitt 62 p.272 claimed to show "in a quite general manner that the quantization of a given system implies also the quantization of any other system to which it can be coupled. By a principle of induction, therefore, the quantum theory must immediately be extended to all physical systems, including the gravitational field". I do not suggest that DeWitt's work is incorrect, but I do not understand it; so it is not discussed here. Zeldovich N 71 p.74 say that "the most general considerations indicate that the gravitational field must obey quantum laws". In particular, the uncertainty principle for electrons, photons, etc. implies that there is a quantum limit on gravitational field measurements (i.e., measurements of the spacetime metric). The authors are not more explicit. This is stronger than what Osborne 49 showed. Exactly the implication which Zeldovich and Novikov claim exists is demonstrated by Eppley H 77. Their demonstration suffers from what I consider to be a grave flaw, though. "The strictly yes or no character and irreversible nature of wave function collapse in quantum measurement theory" (ibid p.54) is an essential requirement for their results. I find such an assumption distasteful and illogical. (A discussion of quantum measurement theory is not possible here. A review is given by Jammer 74.) To summarize, it appears difficult to prove that the gravitational field must be quantized; the meaning of the statement is not clear.

.16 General implications

What are the implications of the ideas discussed in this chapter? Remember figure 9. The operational foundations of GR by itself have only the weakness associated with the use of the continuum in physics. One may regard this weakness as a necessary evil, especially at present, and so not be too concerned about it. Thus GR can be completely adequate for classical physics. When quantum theory is added to GR the operational foundations collapse (gravitationally!). Since quantum theory rests on spacetime theory, the logically simplest resolution of the difficulty is to reject quantum theory and retain GR. Obviously, this should not be done. The alternative is to reject GR (which means GR applied globally, including on the scale of the Planck length), and therefore quantum theory with it, as presently constituted. Whether it is the metric structure or a deeper level which causes the breakdown of the theory one cannot say. (Metric structure seems to be necessary for quantum considerations, as hinted by Ehlers 73 p.37.) Likewise, whether or not a new theory can be found which ascribes a physical menaing to events remains an open question. But there can be no denying that the present theory of space-time and quanta is unsatisfactory. Even if my argument for measurement limitations is not accepted, one cannot approve of GR and quantum theory until they together have good foundations.

.17 Operationalism

Operationalism is, in part, the idea that it must be possible in principle to physically define (that is, to experimentally "pick out") every object which the interpretation rules of a theory says is part of the real world. For example, in quantum mechanics there is no direct physical interpretation attached to the Hilbert space, but it is an essential part of the theory. It need not be operationally defined. On the other hand, a wave field is representative of something real, and it must be possible in principle to physically demonstrate, say the field value at some event, modulo other limitations of the theory (such as the uncertainty principle).

Should it be necessary to defend the implicit belief here in operationalism, I call to my defense Bohr and Rosenfeld, as paraphrased by Marzke W 64 p.48: "every proper theory should provide in and by itself its own means for defining the quantities with which it deals". It has never been shown that quantum theory (including spacetime theory) is "proper", and my contention is that it is not.

The highest aesthetic value with respect to operationalism is held by a theory all the concepts of which are operationally defined. A truly operationally founded theory may not be necessary to account for all observed phenomena, but without such a theory, understanding of nature is necessarily incomplete.

III. Directions to a Better Theory

.0 Initial reason for investigations

Schwinger 58 p.xvi said, in reference to quantum electrodynamics, that "a convergent theory cannot be formulated consistently within the framework of present space-time concepts". It was the divergence problem in quantum field theory and its connection with the concept of event that led to initial attempts to go beyond the GR theory of space-time. More recently, the whole of elementary particle physics has seemed to indicate a need for a different view of space-time. In this chapter I look at some of the ideas which have been advanced in pursuit of such a goal, in relation to the preceding criticisms.

.1 Experimental knowledge

Before setting about constructing a new theory, one has to consider just how adequate the old one is; which of its features are desirable. All of the previous discussion has been theoretical. One should ask the experimentalists for help too! Are there any experimental results which tell us more about the continuum theory, or how it should be changed? There are two important ones. Quantum electrodynamics as applied to certain experiments can be modified by introducing some parameter. The experimental data are then used to place bounds on the actual value of this parameter. For a certain value of the parameter, the original (unmodified) theory is obtained. The experimentally deduced bounds on the parameter can be converted into an upper bound on the size of a region

in which the original theory could be violated. Here is an example from Wilson 72 p.227,231. In the photon propagator, the expression $1/q^2$ is replaced by $1/q^2 \pm 1/(q^2 + k^2)$, corresponding to modifying the Coulomb potential to $1/r(1 \pm e^{-kr/h})$. Experimental data provide a lower bound k_o on k. From the form of the modified Coulomb law, it can be said thet violations of the accepted Coulomb law, $1/r$ (corresponding to the propagator containing $1/q^2$), occur only within distances of h/k_o. The actual experimental results show no violation of quantum electrodynamics and dispersion relation theory, and hence, continuum physics, at scales of the order of 10^{-15} cm. (Hawking E 73 p.57,363; Kirzhnits 67). The same type of approach was used by Kirzhnits C 68. In this case, the Mossbauer effect was studied using Snyder's 47 modification of quantum mechanics. The experimental facts imply that if Snyder's "quantized space-time" theory is correct, the elementary length which it contains must be less than 10^{-20} cm. It is desirable to formulate a version of quantum electrodynamics using Snyder's theory and see what limit experimental knowledge then puts on the elementary length.

One must be careful to not read extra meaning into these statements, as Penrose 75a p.4-5 has pointed out. They do not say that any new space-time theory must have no effects on physics on a scale greater than 10^{-15} cm. They do say that for known phenomena, the present theory is at least a good approximation at such a scale.

.2 Supergravity and superspace

Supergravity is to be a quantum theory of gravity and it has excited some physicists. I think that all of the criticisms of spacetime theory given in II apply to it because it uses conventional spacetime theory and conventional quantum theory (Deser 77).

Misner TW 73 p.1184 say that in the quantum version of superspace theory, spacetime, and hence the usual concept of event, are undefined. Also, the theory is operationally well--founded. In the absence of a formulation of the theory these claims cannot be disputed! However superspace appears not to lead away from the use of the continuum, and I agree with the following by Trautman 72 p.172, which supports a suspicion that superapace is not radical enough in its departure from present theory. "We are not so naive as to try to reduce all phenomena to electromagnetism, but we attempt to model all theories after electrodynamics, classical or quantum ... This may be just what is wrong with what we are doing. General relativity, which is the only other fundamental though classical theory, may play a role in overcoming our prejudices impressed upon us by electrodynamics."

.3 Twistor theory

"The object of twistor theory is to provide a new framework for the mathematical description of basic physics. The conventional picture of a background space-time composed of points is, accordingly, to be replaced by a different one in

which new fundamental entities, the twistors, take over the primary role. At first, twistor theory provides merely a reformulation ... But the new framework suggests different directions in which to proceed, from the ones which might seem natural in a more conventional theory." (Penrose 75a p.1) The desire is to ultimately be able to "build up physical theory from discreteness" (Penrose 71 p.151). A twistor theory reformulation of conventional theory in which twistors have not taken over a primary role is subject to all the criticisms of II. But as Penrose has stated many times (e.g., Penrose 75b p.274-275), in quantum twistor theory, events will not in general be defined.

It is interesting that Einstein 59 p.686 said: "Adhering to the continuum originates with me not in a prejudice, but arises out of the fact that I have been unable to think up anything organic to take its place."

.4 Conclusions and speculations

GR alone is operationally as well-founded as any continuum theory can be. As everyone knows, some of the basic concepts of quantum theory are incompatible with those of GR. Quantum theory can be viewed as a logically consistent theory only when it and its requisite spacetime theory can be constructed in a compatible way.

However, even were that to be achieved, it is very probable that quantum theory would lack a complete operational foundation. In particular, the concept of event seems to be

faulty. More strongly indicated is the possibility that the classical metric structure of spacetime breaks down on the scale of the Planck length. Also, the universality of light propagation may be false.

My work on this essay has led me to the following speculations. Excepting very recent developments perhaps, physics today is done in one of three limiting cases. One is obtained be letting $c \to \infty$; this is non-relativistic quantum theory. Another is the limit $h \to 0$, which is GR. And the third is the neglect of gravity, $G \to 0$, this being quantum field theory. In each of these cases, which are continuum theories, the Planck length reduces to zero, nullifying its real significance. The problem physicists face is to find a correct theory generalizing these limiting cases. I doubt that such a theory will be a continuum theory. Another way of viewing this situation is to consider three large problems in theoretical physics (which are often treated as being unrelated): understanding the nature of particles, divergences (singularities) in GR, and divergences in quantum field theory. It is my opinion that these problems will be solved simultaneously and only when the generalizing theory mentioned above is invented.

May research on these matters lead to more positive conclusions than those presented here!

Reading List

D. Bohm 62 A proposed topological formulation of the quantum theory. in I. J. Good, ed. The Scientist Speculates Basic Books

D. Bohm 66 Space, Time, and the Quantum Theory Understood in Terms of Discrete Structural Processes. in H. Tanikawa, ed. Proceedings of the International Conference on Elementary Particles PTP(*)

H. S. M. Coxeter, G. J. Whitrow 50 World Structure and Non-Euclidean Honeycombs. Proceedings of the Royal Society of London Ser. A 201, 417-437

S. Deser 57 General Relativity and the Divergence Problem in Quantum Field Theory. RMP(*) 29, 417-423

B. S. DeWitt 75 Quantum Field Theory in Curved Spacetime. Physics Reports 19, 295-357

D. Finkelstein 69 Space-Time Code. PR(*) 184, 1261-1271

D. Finkelstein 72 Space-Time Code. III. PR(*) D 5, 2922-2931

R. Fuller, J. A. Wheeler 62 Causality and Multiply Connected Spacetime. PR(*) 128, 919-929

R. P. Geroch 67 Topology in General Relativity. JMP(*) 8, 782-786

R. P. Geroch 72 Einstein Algebras. CMP(*) 26, 271-275

V. L. Ginzburg, D. A. Kirzhnits, A. A. Lyubashin 71 The Role of Quantum Fluctuations of the Gravitational Field in General Relativity Theory and Cosmology. JETP(*) 33, 242-246

R. Gobel 76 The smooth-path topology for curved space-time which incorporates the conformal structure and analytic Feynman tracks. JMP(*) 17, 845-853

S. W. Hawking 75 Particle creation by black holes. CMP(*) 43, 199-220

S. W. Hawking, A. R. King, P. J. McCarthy 76 A new topology for curved space-time which incorporates the causal, differentiable, and conformal structures. JMP(*) 17, 174-181

E. J. Hellund, K. Tanaka 54 Quantized Space-Time. PR(*) 94, 192-195

E. L. Hill 55 Relativistic Theory of Discrete Momentum Space

and Discrete Space-Time. PR(*) 100, 1780-1783

C. W. Isham, R. Penrose, D. W. Sciama, eds. 75 Quantum Gravity Oxford U. P.

E. H. Kronheimer, R. Penrose 67 On the structure of causal spaces. Proceedings of the Cambridge Philosophical Society 63, 481-501

W. Kundt, B. Hoffman 62 Determination Of Gravitational Standard Time. in Recent Developments in General Relativity Pergamon Press

C. A. Mead 66 Observable Consequences of Fundamental-Length Hypotheses. PR(*) 143, 990-1005

R. F. O'Connell 75 Can Quantum Gravitational Forces Stop Gravitational Collapse?. GRG(*) 6, 99-102

R. Penrose, M. A. H. MacCallum 72 Twistor Theory: An Approach to the Quantisation of Fields and Space-Time. Physics Reports 6, 242-315

H. Reichenbach 57 Philosophy of Space and Time Dover Books

V. Rojansky 55 Quantum Mechanical Operators. PR(*) 97, 507

W. B. Rolnick, ed. 74 Causality and Physical Theories American Institute of Physics

H. S. Snyder 47 The Electromagnetic Field in Quantized Space-Time. PR(*) 72, 68-71

P. Yodzis 75 An Algebraic Approach to Classical Spacetime. Proceedings of the Royal Irish Academy 75 A, 37-47

H. Yukawa 66 Atomistics and the Divisibility of Space and Time. Supplement of PTP(*) 37&38, 512-523

(*)
CMP: Communications in Mathematical Physics
GRG: General Relativity and Gravitation
JETP: Soviet Physics, Journal of Experimental and Theoretical Physics
JMP: Journal of Mathematical Physics
PR: Physical Review
PTP: Progress of Theoretical Physics
RMP: Reviews of Modern Physics

Bibliography

S. A. Basri 66 A Deductive Theory of Space and Time North-- Holland

D. I. Blokhintsev 73 Geometry and Physics of the Elementary Particles. in B. Kursunoglu et al, eds. Fundamental Interactions in Physics Plenum

M. Capek 76 The concepts of space and time: their structure and their development D. Reidel

M. A. Castagnino 71 The Riemannian Structure of Space-Time as a Consequence of a Measurement Method. JMP(*) 12, 2203-2211

S. Deser 77 Statement made at GR8 conference, Waterloo, Canada

B. S. DeWitt 62 The Quantization of Geometry. in L. Witten, ed. Gravitation: an introduction to current research John Wiley

J. Ehlers 73 Survey of General Relativity Theory. in W. Israel, ed. Relativity, Astrophysics and Cosmology D. Reidel

J. Ehlers, F. Pirani, A. Schild 72 The Geometry of Free Fall and Light Propagation. in L. O'Raifeartaigh, ed. General Relativity: Papers in Honour of J. L. Synge Clarendon Press

A. Einstein 59 Remarks Concerning The Essays Brought Together In This Co-operative Volume. in P. A. Schlipp, ed. Albert Einstein: Philosopher-Scientist, Vol. II Harper & Bros.

G. F. R. Ellis 75 Cosmology and Verifiability. Quarterly Journal of the Royal Astronomical Society 16, 245-264

K. Eppley, E. Hannah 77 The Necessity of Quantizing the Gravitational Field. Foundations of Physics 7, 51-68

R. Gobel 76 Zeeman Topologies on Space-Times of General Relativity Theory. CMP(*) 46, 289-307

A. Harvey 76 Photon Clocks. GRG(*) 7, 891-893

S. W. Hawking 71 Gravitationally Collapsed Objects of Very Low Mass. Monthly Notices of the Royal Astronomical Society 152, 75-78

S. W. Hawking, G. F. R. Ellis 73 The large scale structure of space-time Cambridge U. P.

W. Heisenberg 38 Uber die in der Theorie der Elementartleichen auftretende universelle Lange. Annalen der Physik 32,

20-33

W. Heisenberg 43 Die "beobachtbaren Grossen" in der Theorie der Elementartleichen I & II. Zeitschrift der Physik 120, 513-538 & 673-702

M. Jammer 74 The Philosophy of Quantum Mechanics John Wiley

D. A. Kirzhnits 67 Upper Limit of Elementary Length. JETP(*) Letters 6, 238-240

D. A. Kirzhnits, V. A. Chechin 68 The Mossbauer Effect and the Theory of Quantized Space-Time. Soviet Journal of Nuclear Physics 7, 275-279

J. R. Klauder, ed. 72 Magic Without Magic: John Archibald Wheeler Freeman

R. F. Marzke, J. A. Wheeler 64 Gravitation as geometry--I: The geometry of space-time and the geometrodynamical standard meter. in H.-Y. Chiu, W. F. Hoffman, eds. Gravitation and Relativity Benjamin

C. A. Mead 64 Possible Connection Between Gravitation and Fundamental Length. PR(*) 135, B849-B862

C. W. Misner, K. Thorne, J. A. Wheeler 73 Gravitation Freeman

F. Murphy 74 The Fundamental Length of Quantized Gravitation. American Journal of Physics 42, 958-959

M. F. M. Osborne 49 Quantum-Theory Restrictions on the General Theory of Relativity. PR(*) 75, 1579-1584

R. Penrose 66 An Analysis of the Structure of Space-Time Unpublished Adams Prize Essay

R. Penrose 68 Structure of Space-Time. in C. DeWitt, J. A. Wheeler, eds. Battelle Rencontres Benjamin

R. Penrose 71 Angular Momentum: An Approach to Combinatorial Space-Time. in T. Bastin, ed. Quantum Theory And Beyond Cambridge U. P.

R. Penrose 72 On the Nature of Quantum Geometry. in Klauder 72

R. Penrose 75a Twistors and Particles: An Outline. Unpublished

R. Penrose 75b Twistor Theory: Its Aims And Achievements. in C. W. Isham, R. Penrose, D. W. Sciama, eds. Quantum Gravity Oxford U. P.

D. H. Perkins 72 Introduction to High Energy Physics Addison--

Wesley

M. Planck 1899 Uber irreversible Strahlungs vorgange. Sitzungber. Deut. Akad. Wiss., Berlin, Kl. Math.-Phys. Tech., 440-480

H. Reichenbach 69 Axiomatization of the Theory of Relativity U. of California Press

H. Salecker, E. P. Wigner 58 Quantum Limitations of the Measurement of Space-Time Distances. PR(*) 109, 571-577

A. Schild 49 Discrete Space-Time and Integral Lorentz Transformations. Canadian Journal of Mathematics 1, 29-47

J. Schwinger, ed. 58 Selected Papers on Quantum Electrodynamics Dover Books

H. S. Snyder 47 Quantized Space-Time. PR(*) 71, 38-41

J. L. Synge 60 Relativity: The General Theory North-Holland

I. E. Tamm 66 On the curved momentum space. in Y. Tanikawa, ed. Proceedings of the International Conference on Elementary Particles PTP(*)

K. S. Thorne 72 Nonspherical Gravitational Collapse: A Short Review. in Klauder 72

A. Trautman 72 Summary of the GR6 Conference. GRG(*) 3, 167-174

D. van Dantzig 56 On the Relation Between Geometry and Physics and the Concept of Space-Time. in Jubilee of Relativity Theory Helvetica Physica Acta Suppl. IV

E. P. Wigner 57 Relativistic Invariance and Quantum Phenomena. RMP(*) 29, 255-268

R. Wilson 72 Lepton-Hadron Interactions and Quantum Electrodynamics. in V. Shelest, ed. Proceedings of the XVth Conference on High Energy Physics Naukova Dumka, Kiev

N. M. J. Woodhouse 73 The differentiable and causal structures of space-time. JMP(*) 14, 495-501

Y. Zeldovich, I. Novikov 71 Relativistic Astrophysics I: Stars and Relativity U. of Chicago Press

(*) See end of reading list.